A Hybrid MMSE Approach to Channel Shortening for Underwater Acoustic OFDM

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Abstract—**Underwater acoustic (UWA) channels pose a significant challenge to multicarrier communication systems. Doppler effects necessitate short block lengths, so that the channel can be approximated as time-invariant, while large delay spreads require long guard intervals to prevent inter-symbol interference. Channel shortening can be used to alleviate this double bind by compressing the received channel and reducing the effective delay spread. Two simple shortening filters are the Time Reversal (TR) and Minimum Mean Squared Error (MMSE) filters. The MMSE filter is a well-studied equalizer, but leads to very long filter lengths. The TR filter is very simple to compute and results in a short filter length, but does not achieve sufficient shortening in sparse UWA channels with a single receive phone. We propose a hybrid approach between these two algorithms, based on the fact that the MMSE filter converges to the TR filter for increasing noise level. By artificially increasing the assumed noise level, we can tune this Hybrid MMSE filter to provide a tradeoff between the performance of the MMSE filter and the simplicity of the TR filter. We perform a series of numerical simulations to show that, for a single-input single-output UWA multicarrier system, the Hybrid MMSE filter can be tuned to provide a shortening filter of fixed length that outperforms both the regular MMSE and TR shortening filters.**

I. Introduction

Underwater acoustic (UWA) channels feature long delay spreads and significant Doppler effects, due to internal waves and both platform and sea-surface motion [1], as well as other dispersive effects that are more difficult to quantify [2]. These long delay spreads motivate the use of multicarrier transmission [3]–[5] that provide low complexity frequency domain equalization based on block transmission and avoid Inter-Symbol Interference (ISI) by using guard intervals between blocks. However, the significant Doppler effects mean that block lengths are limited: they need to be short enough to be able to approximate the channel as time-invariant for the block duration. This means that the product of delay and Doppler spread inherently limits the spectral efficiency of multicarrier communications – if we want to ensure that the guard interval is long enough to encompass the full channel spread and the block length short enough to approximate the channel as time-invariant. One way out of this double bind is channel shortening, it can be used to compress the channel delay spread so that most of its energy fits within a smaller guard interval, thereby improving spectral efficiency.

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In wireline based multicarrier communications like asymmetric digital subscriber line (ADSL), channel shortening has been used successfully [6], [7]. A downside to this approach is that most channel shortening filters need precise knowledge of the channel impulse response and are the result of complex optimization problems, whose complexity increases with the channel delay spread. Some simplified filter designs have been suggested [8], [9], often based on the Minimum Mean-Squared Error (MMSE) filter. As the MMSE filter does not necessarily have a finite impulse response, questions of stability come to mind [10].

Time Reversal (TR) has been previously applied to channel shortening because of its low complexity and short filter length (equal to the delay spread of the channel) [11]–[13]. The drawback to TR is that it mainly works well for rich multipath and when combining multiple receive elements. Unfortunately UWA channels are often very sparse and multiphone combining might be better used to improve bit error rate performance combining symbol estimates [4].

As in multicarrier receivers channel estimation is naturally done in the frequency domain, the complexity of computing the time-domain TR filter is about the same as calculating the standard MMSE filter. Compared to the TR filter, the advantages of an MMSE filter are stronger shortening and accounting for the uncertainty of the channel estimate, but the disadvantage is the (possibly) long filter length. This is even more so a problem as when calculating the MMSE filter based on discrete frequency channel coefficients (as is commonly done), the time-domain is cyclic; therefore if the MMSE filter exceeds the block length used in multicarrier transmission, the time-domain representation will suffer "time-domain aliasing" as it wraps around cyclically.

We are therefore interested in a channel shortening filter that has comparable implementation complexity as TR or MMSE filters *and* can be realized with a finite filter length that is between the guard interval and the block length. The approach that we suggest is based on the observation that the MMSE filter transitions from a close to inverse filter for low noise, with theoretically infinite filter length, to the TR filter with fixed filter length, when the noise tends to infinity. As we wish to limit the filter length for any noise level in the channel estimates, we artificially modify the assumed noise level in the MMSE formulation by adding an extra bias term to achieve a realizable filter length with maximum channel shortening ability.

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We use numerical simulations of an Orthogonal Frequency Division Multiplexing (OFDM) system, to evaluate the performance of this Hybrid MMSE approach. The system uses an OFDM block length of 68 ms and a guard interval of 15 ms. The channels are modeled as time-invariant over a short transmission burst and feature discrete arrivals within a delay spread of up to 35 ms. We find:

- On channels with large ISI the Hybrid MMSE filter can achieve significant gains over both Time Reversal and No Channel Shortening.
- The bias term needs to be chosen as a tradeoff between shorter filter length and sufficient channel shortening.

The remainder of this paper is organized as follows. In Section II, we outline the system model. We discuss channel shortening, and propose our Hybrid MMSE in Section III. Section IV presents the simulation results, with a brief discussion, and we conclude our paper with Section V.

II. Signal Model

A. CP-OFDM

We consider cyclic-prefix (CP) OFDM, as in [14], where in [3]–[5] zero-padded OFDM was used for underwater acoustic communications. Let T denote the OFDM symbol duration and T_{CP} the length of the CP. The total block duration is $T+T_{\text{CP}}$, and the subcarrier spacing is $1/T$. The k-th subcarrier frequency is:

$$f_k = f_c + k/t,\ k = -K/2, \cdots, K/2-1, \tag{1}$$

where f_c is the carrier center frequency and K is the number of subcarriers in use. The bandwidth of this system is $B = K/T$. Let $s[k]$ denote the symbol to be transmitted on the k-th subcarrier. We break the set of K subcarriers into non-overlapping active and null sets, $\mathcal{S}_A$ and $\mathcal{S}_N$, respectively. Thus: $\mathcal{S}_A \cup \mathcal{S}_N = \{-K/2, \cdots, K/2-1\}$. One OFDM block in passband is given by:

$$x(t) = \Re\left\{\left[\sum_{k\in\mathcal{S}_A} s[k]e^{j2\pi\frac{k}{T}t}\right]e^{j2\pi f_c t}\right\}, \tag{2}$$

for $t \in [-T_{\text{CP}}, T+T_g]$.

For simplicity we consider a time-invariant channel model,

$$h(\tau) = \sum_p A_p\,\delta\big(\tau-\tau_p\big). \tag{3}$$

The received signal is than

$$y(t) = x(t) \star h(\tau) + w(t), \tag{4}$$

where $w(t)$ is the ambient noise. At the receiver the signal is down-converted (over-)sampled and fed into a fast Fourier transform (FFT)

$$z_m = \frac{1}{T}\int_0^T y(t)e^{-j2\pi f_c t}e^{-j2\pi\frac{m}{T}t}dt. \tag{5}$$

B. Inter-Symbol Interference

If there is no Inter-Symbol Interference (ISI), i.e., if the maximum delay is less than the CP duration

$$\tau_{\max} := \max_p\{\tau_p\} \leq T_{\text{CP}}, \tag{6}$$

then the output signal on the m-th subcarrier is

$$z_m = H[m]s[m] + w_m. \tag{7}$$

The channel coefficients $H[m]$ are the frequency response of the channel

$$H[m] = \sum_p A_p e^{-j2\pi\left(f_c+\frac{m}{T}\right)\tau_p}, \tag{8}$$

and w_m is modeled as complex Gaussian variables of power N_0.

If some of the delays τ_p exceed the CP length, the section within the $[0,T]$ window will not contain only full cyclic shifts of the transmitted signal $x(t)$. Also the preceding OFDM block will leak into the current window. Both these effects constitute ISI, which we simply model as an increased levels of additive noise $N_0 \to N_0'$. As a measure of the ISI caused by a specific channel realization, we use the ratio of energy beyond the end of the CP duration:

$$\text{ISI} \approx \frac{\int_{T_{\text{CP}}}^{\infty}|h(\tau)|^2\,d\tau}{\int_0^{\infty}|h(\tau)|^2\,d\tau}. \tag{9}$$

Thus, ISI $\in [0,1]$.

III. Hybrid MMSE Shortening

A. TR and MMSE Channel Shortening

The channel shortening filter will be applied in the time-domain, before the OFDM blocks are sliced and fed into the FFT. Given the received signal $y(t)$ and shortening filter $h_{\text{cs}}(\tau)$, the filtered data stream is:

$$\begin{aligned}\widehat{y}(t) &= y(t)\star h_{\text{cs}}(\tau) &(10)\\ &= x(t)\star[h(\tau)\star h_{\text{cs}}(\tau)] + w(t)\star h_{\text{cs}}(\tau). &(11)\end{aligned}$$

This filtered stream is then fed into the FFT block:

$$\widehat{z}_m = \frac{1}{T}\int_0^T \widehat{y}(t)e^{-j2\pi f_c t}e^{-j2\pi\frac{m}{T}t}\,dt. \tag{12}$$

One option for shortening is the use of a Time Reversal (TR) filter. This technique was used in [11]–[13] for MISO and SIMO systems. The single-element time reversal filter is defined in frequency-domain:

$$H_{\text{TR}}[m] = \widetilde{H}^*[m]. \tag{13}$$

TR normally performs well in rich multipath, even with only one phone, and requires a filter no longer than the delay spread of the channel. However, TR with a single phone is not well suited to shortening sparse channels, such as those often found in UWA environments.

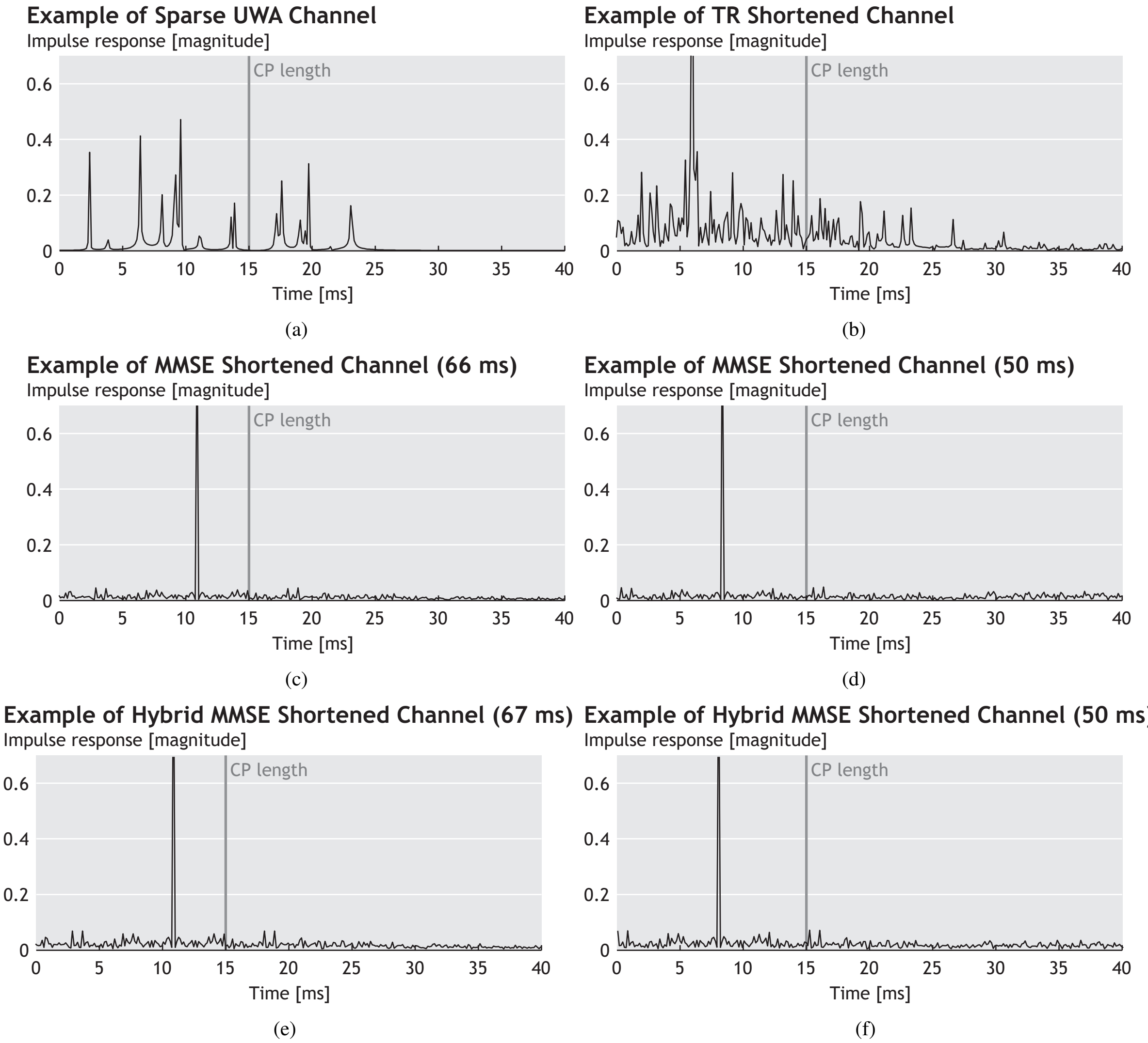


Fig. 1. Example of channel shortening with long and truncated filters. (a) Sample sparse channel with delay spread of 25 ms and cyclic prefix length of 15 ms. (b) Shortened channel resulting from TR filter (13). (c) Shortened channel resulting from MMSE filter (14). (d) Shortened channel resulting from MMSE filter (14) when filter length is limited to 50 ms. (e) Shortened channel resulting from Hybrid MMSE filter (15) with $\delta = 0.1$. (f) Shortened channel resulting from Hybrid MMSE filter (15) with $\delta = 0.1$ when filter is limited to 50 ms.

In the presence of a noisy channel estimate $\widetilde{H}[m]$, the MMSE Inverse is the theoretically optimal shortening filter; defined in frequency-domain:

$$H_{\mathrm{MMSE}}[m] = \frac{\widetilde{H}^*[m]}{\left|\widetilde{H}[m]\right|^2 + \alpha N_0}, \quad m \in \mathcal{S}_A \tag{14}$$

where αN_0 quantifies the channel estimation error in $\widetilde{H}[m]$, and the constant $\alpha < 1$ is a constant that reflects the performance of channel estimation. The corresponding time-domain filter can be (theoretically) infinitely long, but due to the observations being limited to the OFDM subcarriers, any time-domain representation computed from the $\widetilde{H}[m]$ will by cyclical within $[0, T]$. In practice, after the filter impulse response is calculated using an FFT of (13) or (14), we will limit the filter length to be even less than T to reduce filter complexity. This is achieved by simply truncating the impulse response, which does not avoid the aforementioned "time-domain aliasing", since this information is lost as the frequency domain is "sampled".

B. Proposed Hybrid MMSE Channel Shortening

For this reason we propose the Hybrid MMSE shortening filter:

$$H_{\mathrm{H-MMSE}}[m;\delta] = \frac{\widetilde{H}^*[m]}{\left|\widetilde{H}[m]\right|^2 + \alpha N_0 + \delta E_H}, \tag{15}$$

where

$$E_H = \frac{1}{|\mathcal{S}_A|} \sum_{m \in \mathcal{S}_A} \left|\widetilde{H}[m]\right|^2, \tag{16}$$

is the average channel power and $\delta \geq 0$ is a tuning parameter called "bias", which varies the Hybrid MMSE filter between the MMSE filter in (14) and the TR filter in (13). The advantage of adding this bias term in the frequency domain is that it reduces the filter length before the FFT is applied and so can reduce the amount of time-domain aliasing.

Figure 1 shows an example of channel shortening on a sparse channel with a delay spread of 25 ms and cyclic prefix length of 15 ms using the various shortening algorithms we have proposed. The sparse channel we use is shown in Figure 1(a), and has an ISI of 0.1954, as defined in (9). Figure 1(b) shows the result of channel shortening using the TR filter in (13). Since TR is just the time-reversed copy of the channel estimate, the filter in this scenario is only 25 ms long. However, the performance is modest, introducing even more ISI than it removed, raising the ISI slightly to 0.1984 (an extra 0.3% of the channel's energy has been pushed outside of the cyclic prefix). Time Reversal shortening is not well suited to sparse channels with only a single receive element, it performs much better when an array of receivers is linearly combined. Figures 1(c) and 1(d) show the result of channel shortening with the MMSE filter in (14). Figure 1(c) shows the result when the filter is unconstrained, while Figure 1(d) shows the result when the filter length is truncated to 80 ms. In the former case, the ISI has been reduced to 0.0397, while the latter case has an ISI of 0.092. Finally, Figures 1(e) and 1(f) show the shortening results using the Hybrid MMSE shortening filter that we propose in (15), with $\delta = 0.1$. As with the MMSE, the former is an unconstrained filter and yields an ISI of 0.0514, while the latter is truncated to a length of 80 ms and reduces the ISI to 0.0817. While the untruncated MMSE outperforms the untruncated Hybrid MMSE (0.0397 vs. 0.0514), when the filters are truncated to a length of 80 ms, the Hybrid MMSE has a lower ISI value, 0.0877 as compared to 0.092 for the truncated MMSE filter. This illustrates that the Hybrid MMSE is a logical choice for shortening when the filter length is constrained.

In Section IV-A, we will show that the value of δ which minimizes ISI, as defined in (9), is $\delta = 0$ when the filter length in unconstrained, but increases to larger values of δ when the filter length is constrained. The shorter the maximum filter length, the larger the optimal value of δ will be, in terms of ISI. We will further show in Section IV-B that, for certain values of δ, the Hybrid MMSE filter in (15) reduces the impact of ISI on final system performance in terms of uncoded Bit Error Rate (BER) and coded Block Error Rate (BLER).

IV. Simulation Results

We evaluate the performance of channel shortening through numerical simulations, under the assumption of a sparse UWA channel. We consider $N_p = 15$ discrete paths, where the inter-arrival times are exponentially distributed with mean $E[\tau_{p+1} - \tau_p] = 3$ ms. Hence, the average channel delay spread is roughly 45 ms. We truncate the generated channels at 35 ms, though, to ensure that our $T = 68$ ms preamble contains enough information to estimate the channel. The amplitudes are Rayleigh distributed with the average power decreasing exponentially with delay, where the difference between the beginning and end of the guard time of 15 ms is 6dB.

TABLE I
ZP-OFDM Parameters in Numerical Simulation

Parameter	Symbol	Value
carrier frequency	f_c	30 kHz
bandwidth	B	7.5 kHz
no. subcarriers	K	512
symbol duration	T	68 ms
subcarrier spacing	$1/T$	14.65 Hz
cyclic prefix duration	T_{CP}	15 ms

The carrier frequency, bandwidth, number of subcarriers, inter carrier spacing, and symbol interval are summarized in Table I. Out of a total $K = 512$ subcarriers, $|\mathcal{S}_N| = 51$ are null subcarriers at the two ends of the spectrum. We do not use any pilot symbols in this test because the channel is time-invariant, and can be estimated from the preamble. The remaining $|\mathcal{S}_A| = 461$ subcarriers are encoded using a rate 1/2 convolutional code of constraint length 9. With a QPSK constellation, the spectral efficiency η and rate R are:

$$\eta = \frac{T}{T + T_{\text{CP}}} \cdot \frac{461}{512} \cdot \frac{1}{2} \cdot \log_2 4 = 0.74 \,\text{bits/s/Hz}, \quad (17)$$
$$R = \eta B = 5.53 \,\text{kb/s}. \quad (18)$$

Each transmission consists of a preamble as described in [15], followed by 20 OFDM blocks. The channel is assumed constant during this duration and we repeat this test for 20 randomly generated channels, yielding a total test bank of 400 OFDM blocks.

A. Bias and ISI Gain

To test the behavior of δ, we ran a set of simulations with varying filter lengths and swept δ over the values $\delta \in \{0.1, 0.5, 1, 10\}$. The filter lengths that we chose range from 16.6 ms, just longer than the cyclic prefix of the channel, to 66.7 ms, slightly less than the OFDM block length of 68 ms. In between, we have sample points at 33.3 ms and 50 ms as well. The former corresponds roughly to the delay spread of the simulated channels. We then compared those results to similar tests for time reversal shortening and no shortening at all.

Figure 2 plots the results of this set of simulations. We set the signal-to-noise ratio to 20 dB and utilize sparse channel estimation as in [5] to generate $\widetilde{H}(\omega)$. This is necessary as based on the very large delay spread, a single OFDM block as preamble would lead to channel estimation error on the order of the additive noise $\alpha \approx 1$ for a conventional least-squares estimator, as the number of unknowns is comparable to the number of observations. First note that under no shortening, the ISI is just over 0.22. Time Reversal, noticeably, does not reduce the ISI at any of these filter lengths. The Hybrid MMSE filters, however, do perform quite well. At shorter filter lengths (16.6 ms and 33.3 ms), the smallest ISI potential is achieved with larger δ values (10 and 1, respectively), while the optimal value is $\delta = 0.5$ at 66.7 ms. This confirms our argument that

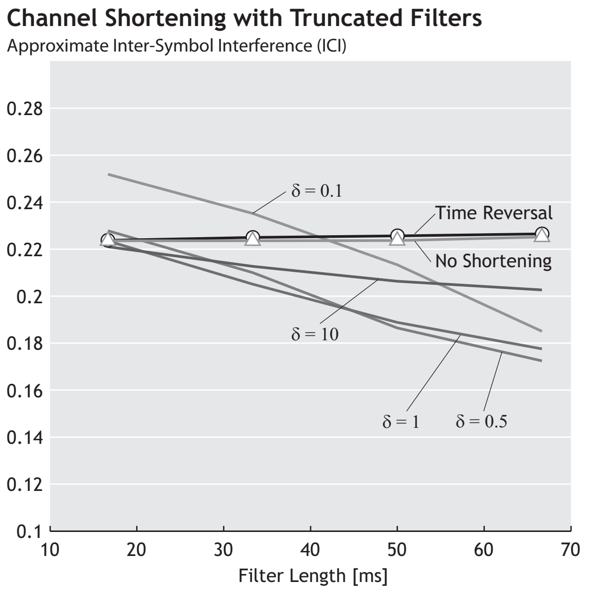


Fig. 2. ISI estimates for various shortening filters when the signal-to-noise ratio is 20 dB.

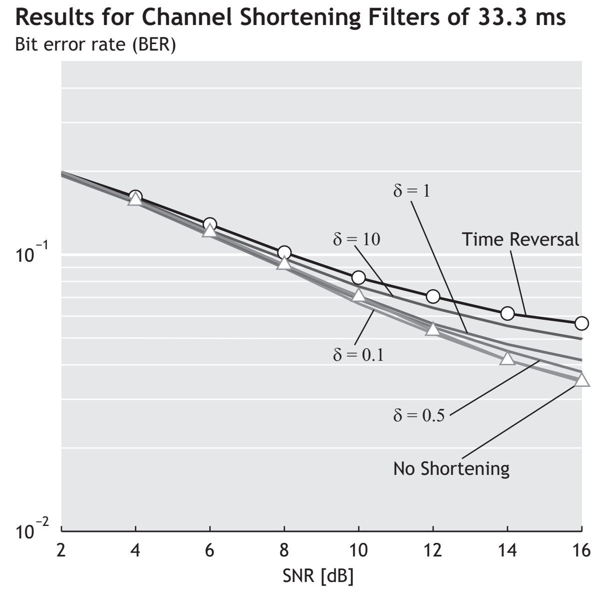
the optimal δ will depend on the desired filter length and that shorter filters favor larger values of δ.

B. Performance Results

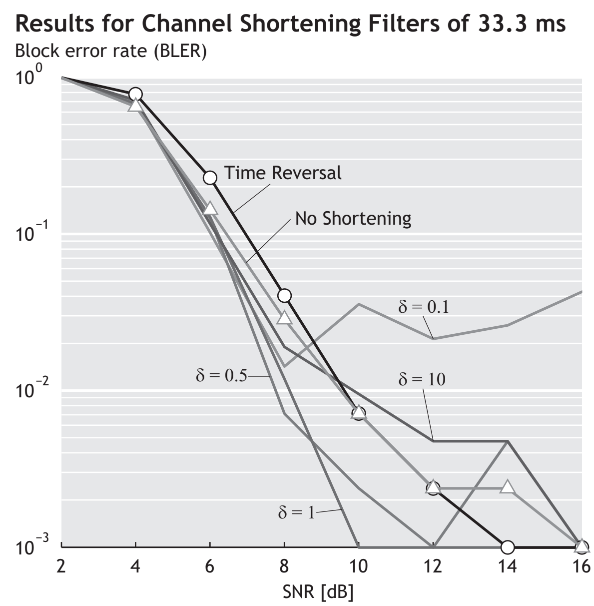
We now present Bit Error Rate (BER) results to quantify the performance of our Hybrid MMSE shortening filter. In addition to the raw BER, which is measured after demodulation, we will also make use of the Block Error Rate (BLER). The BLER is a quantized form of decoded BER; it is the fraction of blocks that contain one or more bit errors after error correction. We ran the test for three different filter lengths: 33 ms, 50 ms, and 66 ms. The first is roughly equal to the delay spread of the channel, the second is just less than twice the delay spread, and the third is roughly equivalent to the length of one ODFM block (68 ms).

The raw BER curves for the three tests are shown in Figures 3(a), 4(a), and 5(a). The first thing to note is that, for all three tests, the Hybrid MMSE filter favors smaller values of δ, with $\delta = 1$ performing equivalently to no shortening, and Time Reversal performing the worst. This result is limited to raw BER, and is not (necessarily) indicative of the decoded BLER results.

The BLER curves for the three tests are shown in Figures 3(b), 4(b), and 5(b). We first note that, while Time Reversal consistently performed worse than no shortening before, here we can see slight gains, especially at 50 ms. The fact that the BLER and raw BER results seem contradictory can be explained this way: for channel realizations with low ISI the TR filter adds more ISI than it removes, while for the channels with heavy ISI it seems beneficiary. As a curiosity the performance of TR degrades slightly at 66 ms, although no channels are longer than 50 ms and therefore the results should be the same. We speculate that channel estimation can lead to



(a)



(b)

Fig. 3. Plots of the (a) uncoded BER and (b) coded BLER, for SNR=2-16 dB with various shortening algorithms with a maximum filter length of 33 ms.

spurious channel tap detections outside the 50 ms that will be included in the TR filter for 66 ms filter length, increasing ISI.

Looking at the 33.3 ms case, we see all of the Hybrid MMSE filters perform similarly to the no shortening case when SNR is very low (4 dB). As SNR increases, though, the Hybrid MMSE filters all begin to shorten the channel, with $\delta = 0.5$ and $\delta = 1$ performing the best. When we move to the mid-range filter, 50 ms, we see that the optimal Hybrid MMSE filter now has a value of $\delta = 10$, showing the effect of filter length on the performance of different δ parameters. In this case, all four of the tested Hybrid MMSE filters (and TR) decoded all blocks correctly at 12 dB SNR, but the best filters achieve

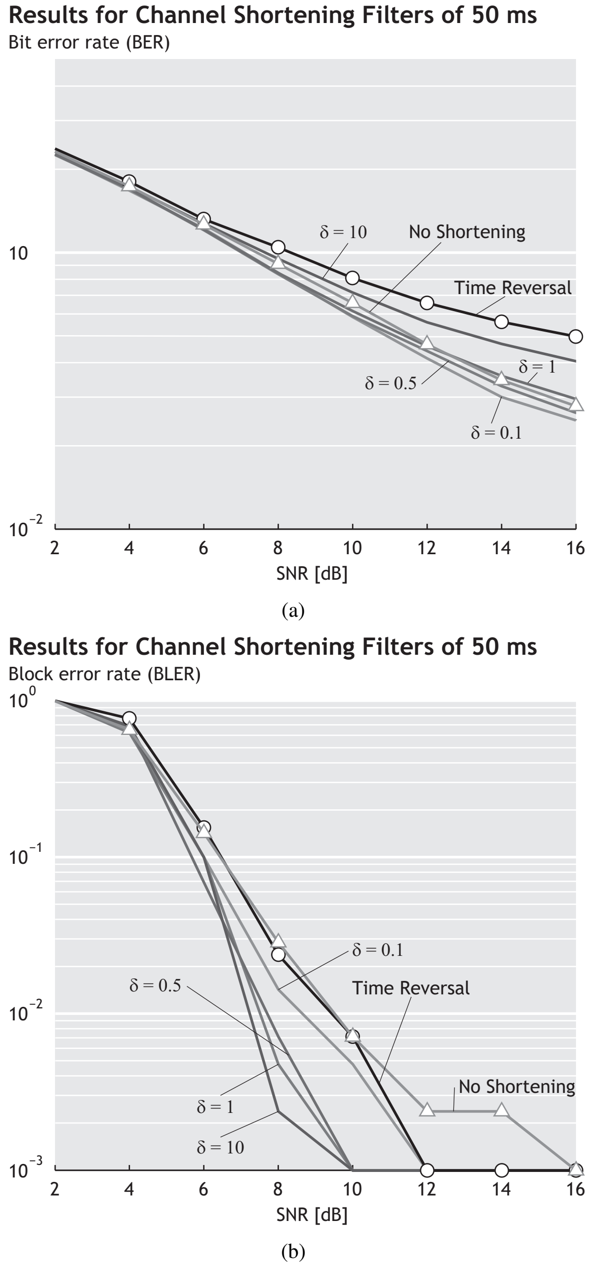


Fig. 4. Plots of the (a) uncoded BER and (b) coded BLER, for SNR=2-16 dB with various shortening algorithms with a maximum filter length of 50 ms.

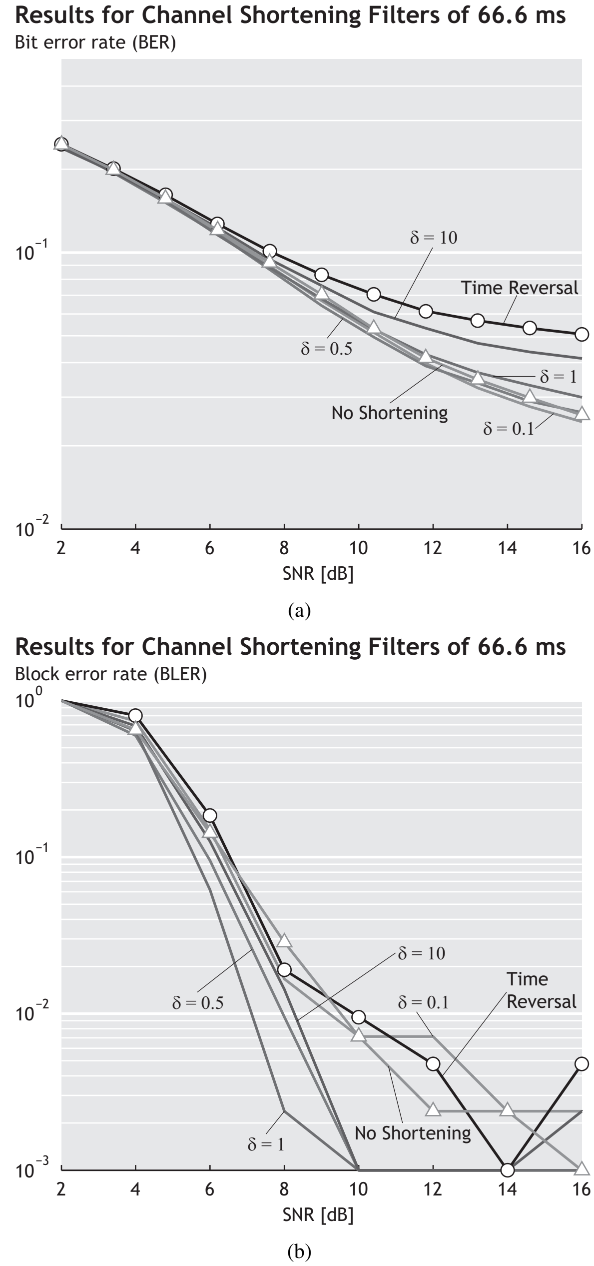


Fig. 5. Plots of the (a) uncoded BER and (b) coded BLER, for SNR=2-16 dB with various shortening algorithms with a maximum filter length of 66 ms.

this level a full 2 dB before. In comparison without shortening one block remains in error at 12 dB, while the best Hybrid MMSE achieve this already at 8 dB. Similar to the 50 ms test, the 66.6 ms test shows the filters with $\delta = 0.5, 1,$ and 10 outperforming the no shortening case. From these results, it can be shown that values of δ between 0.5 and 1 are suitable choices for the set of sparse channels that we simulated.

V. Conclusion

We have presented here a new algorithm for channel shortening, the Hybrid MMSE shortening filter. This filter is a tunable compromise between the TR and MMSE shortening filters. We have shown through numerical simulations that the optimal value of the H-MMSE parameter δ depends on the desired filter length. BER and BLER curves have shown that the Hybrid MMSE shortening filter can significantly improve performance in the presence of long delay spreads.

References

[1] T. H. Eggen, A. B. Baggeroer, and J. C. Preisic, "Communication over Doppler spread channels, Part I: Channel and receiver presentation," *IEEE. J. Ocean. Eng.*, vol. 25, no. 1, pp. 62–71, Jan. 2000.

[2] M. Stojanovic and J. Preisic, "Underwater acoustic communication channels: Propagation models and statistical characterization," *IEEE Comm. Mag*, vol. 47, no. 1, pp. 84–89, Jan. 2009.

[3] M. Stojanovic, “Low complexity OFDM detector for underwater channels,” Boston, MA, Sept. 18-21, 2006.
[4] B. Li, S. Zhou, M. Stojanovic, L. Freitag, and P. Willett, “Multicarrier communication over underwater acoustic channels with nonuniform Doppler shifts,” vol. 33, no. 2, pp. 198–209, Apr. 2008.
[5] C. R. Berger, S. Zhou, J. Preisig, and P. Willett, “Sparse channel estimation for multicarrier underwater acoustic communication: From subspace methods to compressed sensing,” vol. 58, no. 3, pp. 1708–1721, Mar. 2010.
[6] P. Melsa, R. C. Younce, and C. E. Rohrs, “Impulse response shortening for discrete multitone transceivers,” *IEEE Trans. Communications*, vol. 44, no. 12, pp. 1662–1672, Dec. 1996.
[7] N. Al-Dhahir and J. M. Cioffi, “Optimum finite-length equalization for multicarrier transceivers,” *IEEE Trans. Communications*, vol. 44, no. 1, pp. 56–64, Jan. 1996.
[8] A. Tkacenko and P. Vaidynathan, “A low-complexity eigenfilter design method for channel shortening equalizers for DMT systems,” *IEEE Trans. Communications*, vol. 51, no. 7, pp. 1069–1072, Jul. 2003.
[9] D. Daly, C. Heneghan, and A. D. Fagan, “Minimum mean-squared error impulse response shortening for discrete multitone transceivers,” *IEEE Trans. Signal Proc.*, vol. 52, no. 1, pp. 301–306, Jan. 2004.
[10] R. López-Valcare, “Realizable minimum mean-squared error channel shorteners,” *IEEE Trans. Signal Proc.*, vol. 53, no. 11, pp. 4354–4362, Nov. 2005.
[11] D. Rouseff, D. R. Jackson, W. L. J. Fox, C. D. Jones, J. A. Ritcey, and D. R. Dowling, “Underwater acoustic communication by passive-phase conjugation: Theory and experimental results,” *IEEE J. Oceanic Eng.*, vol. 26, no. 4, pp. 821–831, October 2001.
[12] H. C. Song, S. Kim, W. S. Hodgkiss, and W. A. Kuperman, “Environmentally adaptive reverberation nulling using a time reversal mirror,” *J. Acoust. Soc. Am.*, vol. 116, no. 2, pp. 762–768, August 2004.
[13] P. Blomgren, P. Kyritsi, A. D. Kim, and G. Papanicolaou, “Spatial focusing and intersymbol interference in Multiple-Input-Single-Output Time Reversal communication systems,” *IEEE J. Oceanic Eng.*, vol. 33, no. 3, pp. 341–355, 2008.
[14] C. R. Berger, J. Gomes, and J. M. F. Moura, “Study of pilot designs for cyclic-prefix OFDM on time-varying and sparse underwater acoustic channels,” Santander, Spain, Jun. 2011. [Online]. Available: http://www.ece.cmu.edu/∼crberger
[15] S. Mason, C. R. Berger, S. Zhou, and P. Willett, “Detection, synchronization, and Doppler scale estimation with multicarrier waveforms in underwater acoustic communication,” vol. 26, no. 9, pp. 1638–1649, Dec. 2008.