SINGLE ANTENNA TIME REVERSAL DETECTION OF MOVING TARGET

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ABSTRACT

This paper is concerned with a moving target detection using time reversal in dense multipath environments. We show that the Doppler shift in the time reversal re-transmission simplifies the detector design, yet still achieves the focusing effect. Thus, the Doppler diversity is utilized to achieve high target detectability by time reversal.

Index Terms— Time Reversal, Doppler, Detection

1. INTRODUCTION

Multipath is a common physical phenomenon in radar, sonar and wireless communication applications. For example, the development of underwater sonar and communication systems must cope with challenging environmental conditions, such as severe multipath due to sound reflection on the surface and bottom. For radar systems, the urban scenario is rich in multipath propagation generated by multiple reflections, refractions, and scattering of the radar signal from buildings and other structures. Time reversal has been explored as a complement to conventional multipath compensation [1]. We have developed time-reversal based data processing algorithms for target detection and localization [2]. In addition to the theoretical analysis of time reversal, experiments have been conducted in underwater acoustics and electromagnetic to demonstrate the focusing effect of time reversal [3, 4]. In a typical time reversal experiment, the signal received by an antenna would be digitized, energy normalized, time-reversed, and retransmitted from the same antenna element. The underlying condition for time reversal is that the propagation media is reciprocal. Under this condition, time reversal achieves spatial and temporal focusing of energy. In reality, the source, the antennas, and the medium may not be strictly stationary. Questions rise if the time reversal focusing is still achievable when either the medium or the source is non-stationary. Underwater acoustic experiments and theoretical analysis have demonstrated that the focusing can still be achieved [1, 3, 5]. In this paper, we address the problem of time reversal detection of moving target using a bi-static radar. For simplicity, we assume that the target moves, while the channel and the antennas are stationary. In this paper, we analyze the impact of Doppler shift due to target motion in a dense multipath environment on time reversal focusing, and subsequently the detector design and performance.

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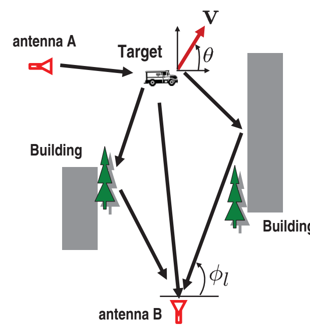

Fig. 1. Moving target detection in dense multipath environment using a bi-static radar. Antenna A sends a probe signal; Antenna B acts like a mirror by re-sending the time-reversed received signal; Detection is implemented by Antenna A.

2. PROBLEM DESCRIPTION AND SIGNAL MODEL

2.1. Multipath Environments

We assume that the background environment is stationary and can be probed before the target appears. This assumption implies that the dominant multipath due to the background can be identified by successive probing and then subtracted from the test data that may contain the reflections from a moving target. This assumption simplifies the problem and our analysis. In a multipath rich environment, for example, urban environments, building walls produce specular reflections of the radar signal, which impinge on the target from different incident angles. Thus, multipath propagation increases the spatial diversity of the radar system. Furthermore, each multipath component is affected by a different Doppler shift corresponding to the projection of the target velocity on the direction-of-arrival (DOA) of each path. The Doppler frequency shift is widely used to separate moving targets from stationary clutter [6]. Exploitation of multiple Doppler shifts should increase the ability of the radar to detect targets. In this paper, we discuss the impact of target motion on time reversal based detectors.

2.2. Signal Model

We consider, for example, a ground moving target in a urban canyon environment rich in multipath reflections as illustrated in Fig. 1. Let $s(t)$ represent the transmit signal from the radar antenna with total energy $E_s = \int_0^T s(t)^2 dt$, where T is the signal duration. This signal impinges on the target; the reflected signal from the target, as well as from the surrounding reflectors, reaches the receive antenna. Let $h_l(t)$ denote the l-th propagation path channel impulse response between the transmitter, the target, and the receiver. The target is assumed to be in the far-field and is moving with a constant velocity $\mathbf{v} = (|\mathbf{v}|\cos\theta, \ \ |\mathbf{v}|\sin\theta)$ relative to the radar, where θ is the relative angle. Since L multipath signals impinge on the target from different incident angles, they produce L different relative Doppler shifts. We let $\mathbf{u}_l = (\cos\phi_l, \sin\phi_l)$ denote the unit vector of the direction of arrival (DOA) along the l-th propagation path, where

$$\phi_l \in [0, \ 2\pi], \quad l = 0, \cdots, L-1. \tag{1}$$

Therefore, the scaling factor of the differential Doppler shift for the l-th path can be written as

$$\beta_l = \frac{\langle \mathbf{v}, \mathbf{u}_l \rangle}{c} = \frac{|\mathbf{v}|(\cos\theta\cos\phi_l + \sin\theta\sin\phi_l)}{c}, \tag{2}$$

where c is the speed of propagation; $\langle \cdot, \cdot \rangle$ denotes the inner-product operator over the real vector space. Thus, the multipath propagation provides enhanced spatial diversity to a radar system. Next, we describe the signal model of forward transmission and time reversal backward retransmission.

2.2.1. Initial Probing - Forward Transmission

Antenna A transmits a probe waveform $s(t)$, the received l-th path signal reflected from the target is (ignore noise for the moment) is recorded at the antenna B.

$$\begin{aligned} x_l(t) &= \int h_l(\tau) s(t-\tau) e^{j\omega_c \beta_l (t-\tau)} d\tau & (3) \\ &= s(t) e^{j\omega_c \beta_l t} * h_l(t), & (4) \end{aligned}$$

where the symbol $*$ represents the convolution. Eqn. (4) shows that the pulse shape $s(t)$ is distorted by the l-th Doppler shift. Next, let

$$h(t) = \sum_{l=0}^{L-1} h_l(t), \ \ 0 \le t \le T_s \tag{5}$$

denote the time dispersive channel response; $h(t)$ is a time spreading function of effective length T_s. To be more specific, $h_l(t)$ is defined as

$$h_l(t) = \alpha_l \delta(t - \tau_l). \tag{6}$$

Hence, the total received signal from the forward transmission is given by

$$\begin{aligned} x(t) &= \sum_{l=0}^{L-1} x_l(t) + v(t) & (7) \\ &= \sum_{l=0}^{L-1} \alpha_l s(t-\tau_l) e^{j\omega_c \beta_l (t-\tau_l)} + v(t) & (8) \end{aligned}$$

where $v(t)$ is the additive Gaussian noise and is independent of the signal; L is the number of dominant paths.

2.2.2. Time Reversal - Backward Retransmission

In the backward transmission, the antenna B acts as a time reversal mirror. Because this is a single antenna that acts like a mirror, this scenario represents the limiting case for time reversal. Based on the assumption we made in 2.1, the signals due to background can be subtracted. The residue received signal $x(t)$ is time-reversed, energy normalized, and re-sent. We assume that the delay between the forward and time-reversed transmissions is sufficiently small so that the target displacement during that interval will be negligible, and the multipath structure of the channel can be regarded as constant. The purpose of time reversal re-transmission is to utilize the spatial and temporal focusing of time reversal to compensate for the multipath. The target detection will be implemented at the original antenna location, i.e., at the antenna A. The received signal at Antenna A becomes

$$\begin{aligned} y(t) &= \sum_{l=0}^{L-1} h_l(t) * k x^*(-t) e^{j\omega_c \beta_l t} & (9) \\ &= k \sum_{l=0}^{L-1} h_l(t) * \left(\sum_{l'=0}^{L-1} s^*(-t) e^{j\omega_c \beta_{l'} t} * h_{l'}^*(-t) \right) \\ &\quad e^{j\omega_c \beta_l t} \\ &= k \sum_{l=0}^{L-1} \sum_{l'=0}^{L-1} \alpha_l \alpha_{l'}^* s^*(-t + \tau_l - \tau_{l'}) e^{j\omega_c \beta_{l'} (-t + \tau_l - \tau_{l'})} \\ &\quad e^{j\omega_c \beta_l (t - \tau_l)} \\ &= k \sum_{l=0}^{L} |\alpha_l|^2 s^*(-t) e^{-j\omega_c \beta_l \tau_l} + & (10) \\ &\quad k \sum_{l \neq l'}^{L-1} \alpha_l \alpha_{l'}^* s^*(-t + \tau_l - \tau_{l'}) e^{j\omega_c \beta_{l'} (-t + \tau_l - \tau_{l'})} \\ &\approx k \sum_{l=0}^{L} |\alpha_l|^2 s^*(-t) e^{-j\omega_c \beta_l \tau_l} & (11) \\ &= k \, G_L s^*(-t) & (12) \end{aligned}$$

where

$$\begin{aligned} G_L &\triangleq \sum_{l=0}^{L} |\alpha_l|^2 e^{-j\omega_c \beta_l \tau_l} & (13) \\ k &\triangleq \sqrt{\frac{E_s}{\int_0^{T_s} |x(t)|^2 dt}} & (14) \end{aligned}$$

In the above derivation, we use Eqn. (6) to represent a multipath channel. The approximation in (11) is valid because of the focusing effect in time reversal re-transmission. The scaling factor k is to ensure the same amount of transmission energy. Eqn. (13) is the total gain of the L paths, each path has its own Doppler shift. It implies that multipath compensation is achieved by time reversal even in the presence of differential Doppler shifts, however, the focus gain is modulated by phase rotation. To further quantify the focusing gain in the presence of the Doppler shift, we analyze the term G_L by statistical approaches. Note that the phase term

$$\theta_l \triangleq \omega_c \beta_l \tau_l = 2\pi f_c \beta_l \tau_l \tag{15}$$

will change by 2π rad whenever $\beta_l \tau_l$ changes by $\frac{1}{f_c}$, where f_c is the radar carrier frequency. But $\frac{1}{f_c}$ is a small number and, hence, θ_l can change by 2π rad with relatively small motions of the target or a propagation path. Because the number of multipaths L is assumed to be large, the impinging angle of the path to the antenna is assumed to be $[-\pi, \pi]$ in an unpredictable (random) manner. This implies that G_L can be modeled as a random process. When there are a large number of paths, the central limit theorem applies. This means G_L is a complex valued Gaussian random process,

$$G_L \simeq \mathcal{CN}\left(\sum_{l=1}^{L} \mu_l, \sum_{l=1}^{L} \Phi_l\right), \tag{16}$$

where

$$\begin{aligned} \mu_l &= E\{|\alpha_l|^2 e^{j\omega_c \beta_l \tau_l}\} = 0, & (17)\\ \Phi_l &= Var\{|\alpha_l|^2 e^{j\omega_c \beta_l \tau_l}\} = Var\{|\alpha_l|^2\} & (18) \end{aligned}$$

Here we assume that the l-th path gain α_l is a complex Gaussian random variable with zero mean and variance σ_α^2. Hence, $|\alpha_l|^2$ is distributed as a $\sigma_\alpha^2 \chi_2^2$, where χ_2^2 denotes the central chi-squared distribution with 2 degrees of freedom. Therefore, $\Phi_l = 4\sigma_\alpha^4$, i.e., (16) can be re-written as

$$G_L \simeq \mathcal{CN}\left(0, 4L\sigma_\alpha^4\right), \tag{19}$$

3. DETECTORS

In this section, we formulate the time reversal detection problem for moving target where the Doppler shift is present. To emphasize that we have a multipath rich environment in which time reversal achieves full advantage, we make the following assumptions: (A1) The number of paths L is sufficiently large; (A2) The multipath direction of arrival $0 \leq \phi_l \leq 2\pi$ is uniformly random; and (A3) The multipath complex gain is Gaussian distributed with zero mean and variance σ_α^2, i.e., $\alpha_l \sim \mathcal{CN}(0, \sigma_\alpha^2)$.

3.1. Time reversal detector

From Eqn. (12), we know that the received signal after time-reversal re-transmission can be written as follow:

$$y(t) = z(t) + w(t), \;\; 0 \leq t \leq T \tag{20}$$

where $w(t)$ is the additive noise, T is the observation time of the signal, and

$$z(t) \triangleq k\, G_L s^*(-t). \tag{21}$$

Finally, we assume that the received signal is converted to baseband prior to sampling and then sampled at sampling rate $f_s = B$. The discrete version of (20) is

$$y(n) = z(n) + w(n), \;\; n = 0, \cdots N-1 \tag{22}$$

where $N = f_s T$. Define

$$\begin{aligned} \mathbf{y} &= [y(0), y(1), \cdots, y(N-1)]^T & (23)\\ \mathbf{z} &= [z(0), z(1), \cdots, z(N-1)]^T & (24)\\ \mathbf{w} &= [w(0), w(1), \cdots, w(N-1)]^T & (25) \end{aligned}$$

Using (19), we have

$$z(n) \sim \mathcal{CN}\left(0, k^2 4L\sigma_\alpha^4 |s(-n)|^2\right). \tag{26}$$

Here we assume that the scalar k is approximately a constant. This assumption is approved to be valid [2]. Hence, the time-reversal detection problem in the presence of target motion is

$$\begin{aligned} \mathbb{H}_1: \quad \mathbf{y} &= \mathbf{z} + \mathbf{w} \\ \mathbb{H}_0: \quad \mathbf{y} &= \mathbf{w} \end{aligned} \tag{27}$$

3.2. Conventional detector

The discrete form of the received signal from the forward transmission, (8), can be written as

$$\begin{aligned} x(n) &= \sum_{l=0}^{L-1} \alpha_l s(n-l) e^{j\omega_c \beta_l (n-l)/f_s} + v(n), & (28)\\ &= r(n) + v(n) & (29) \end{aligned}$$

where $n = 0, \cdots, N-1$, $L = f_s T_s$, and

$$\begin{aligned} r(n) &\triangleq \sum_{l=0}^{L-1} \gamma_{n,l} s(n-l) & (30)\\ \gamma_{n,l} &\triangleq \alpha_l e^{j\omega_c \beta_l (n-l)/f_s} \sim \mathcal{CN}(0, \sigma_\alpha^2). & (31) \end{aligned}$$

We notice that, in the absence of Doppler shift (i.e., if $\beta_l = 0$), the likelihood ratio test detector would be the replica correlation integration detector given in [7]. In the presence of the Doppler shift, and given the assumptions (A1) and (A2), we will employ the central limit theorem to simplify the detector design. From (30) and (31), by the central limit theorem, we obtain

$$r(n) \simeq \mathcal{CN}\left(\sum_{l=1}^{L} \eta_l, \sum_{l=1}^{L} \Psi_l\right), \tag{32}$$

where

$$\begin{aligned} \eta_l &= E\{\gamma_{n,l} s(n-l)\} = 0, & (33)\\ \Psi_l &= Var\{\gamma_{n,l} s(n-l)\} = \sigma_\alpha^2 |s(n-l)|^2 & (34) \end{aligned}$$

Therefore, the statistics for $r(n)$ can be written as

$$r(n) \simeq \mathcal{CN}\left(0, \sigma_\alpha^2 \sum_{l=0}^{L-1} |s(n-l)|^2\right), \tag{35}$$

Define

$$\begin{aligned} \mathbf{x} &= [x(0), x(1), \cdots, x(N-1)]^T & (36)\\ \mathbf{r} &= [r(0), r(1), \cdots, r(N-1)]^T & (37)\\ \mathbf{v} &= [v(0), v(1), \cdots, v(N-1)]^T & (38) \end{aligned}$$

which leads to the conventional detection problem formulated as

$$\begin{aligned} \mathbb{H}_1: \quad \mathbf{x} &= \mathbf{r} + \mathbf{v} \\ \mathbb{H}_0: \quad \mathbf{x} &= \mathbf{v} \end{aligned} \tag{39}$$

4. PERFORMANCE ANALYSIS AND SIMULATION

In this section, we will analyze the performance of the TR detector. The signal-to-noise ratio is defined as $\mathrm{SNR} = 10 \cdot \log_{10} \frac{E_s \sigma_\alpha^2}{\sigma_v^2}$. To derive the data statistics, we first consider the noise-only case under $\mathbb{H}_0$. For the detection problem (27), the data is distributed as $\mathbf{y} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I})$; For the detection problem (39), the data is distributed as $\mathbf{x} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I})$. Under the signal plus noise case (i.e., $\mathbb{H}_1$), for the detection problem (27), the data is distributed as

$$\mathbf{y} \sim \mathcal{CN}\left(0, k^2 L \sigma_\alpha^4 \mathbf{s}^* \mathbf{s}^T + \sigma_n^2 \mathbf{I}\right). \tag{40}$$

where $\mathbf{s} = [s(0), s(1), \cdots, s(N-1)]^T$. For the detection problem (39), the data is distributed as

$$\mathbf{x} \sim \mathcal{CN}\left(0, \mathbf{\Psi} + \sigma_n^2 \mathbf{I}\right) \tag{41}$$

where $\mathbf{\Psi} = \mathrm{diag}[\psi_0, \cdots, \psi_{N-1}]$, and

$$\psi_i = \frac{1}{\sigma_n^2 + \sigma_\alpha^2 \sum_{l=0}^{L-1} |s(i-l)|^2}, \quad i = 0, \cdots, N-1 \tag{42}$$

We notice that (27) or (39) can be formulated as the Gauss-Gauss problem where both the signal and the noise have Gaussian distributions [8]. Hence, the detector can be derived using the likelihood ratio test

$$\ell(\mathbf{x}) = \log \frac{f(\mathbf{x}; \mathbb{H}_1)}{f(\mathbf{x}; \mathbb{H}_0)}, \tag{43}$$

where $f(\cdot; \mathbb{H}_i)$ is the probability density function under $\mathbb{H}_i$, $i = 0, 1$, respectively. Due to space limitation, we present here the final test statistic and omit the detailed mathematical derivation. The time reversal detector is

$$\ell_{\mathrm{TR}}(\mathbf{y}) = \left| \sum_{n=0}^{N-1} y(n) s(-n) \right|^2. \tag{44}$$

The conventional detector is

$$\ell_{\mathrm{CON}}(\mathbf{x}) = \sum_{l=0}^{L-1} \sum_{n=0}^{N-1} \frac{|x(n) s^*(n-l)|^2}{\sigma_n^2 + \sigma_\alpha^2 \sum_{l=0}^{L-1} |s(n-l)|^2} \tag{45}$$

Next, we conduct numerical simulations. The transmitting signal is a linear frequency modulated (LFM) signal (in baseband)

$$s(t) = e^{j 2\pi B/(2T) t^2}, \quad -T/2 \le t \le T/2 \tag{46}$$

where $B = f_s = 20$ MHz, $T = 5\ \mu$ second. Assume that the channel dispersion is $T_s = 1\ \mu$ second, $\sigma_\alpha^2 = 2$. Hence, $N = f_s T = 100$, and $L = f_s T_s = 20$. Fig. 2 shows the receiver operating characteristic (ROC) curve at the false alarm rate of $P_{\mathrm{FA}} = 10^{-4}$ using $300,000$ Monte Carlo runs. The result shows that the time reversal method has about 2.5-dB gain over the conventional detector at $P_d = 0.8$.

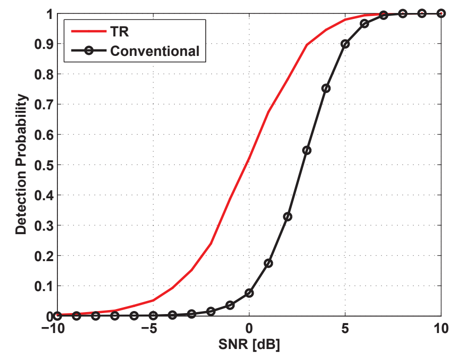

Fig. 2. Receiver operating characteristics. The number of dominant paths $L = 20$; The false alarm rate $P_{\mathrm{FA}} = 10^{-4}$.

5. CONCLUSION

In this paper, we develop a time reversal detector for moving target. Our analysis shows that time reversal achieves spatial focusing in the presence of Doppler shifts. Due to target motion, the focusing gain is modulated with a phase rotation. In dense multipath, the time reversal detector shows a significant performance gain over the conventional detector.

6. REFERENCES

[1] D. R. Jackson and D. R. Dowling, "Phase conjugation in underwater acoustics," *Journal of Acoustical Society of America*, vol. 89, no. 1, pp. 171–181, January 1990.

[2] Y. Jin and J. M. F. Moura, "Time reversal detection using antenna arrays," *IEEE Transactions on Signal Processing*, vol. 57, no. 4, pp. 1396–1414, April 2009.

[3] W. J. Higley, P. Roux, W. A. Kuperman, W. S. Hodgkiss, H. C. Song, T. Akal, and M. Stevenson, "Synthetic aperture time-reversal communications in shallow water: Experimental demonstration at sea," *Journal of Acoustical Society of America*, vol. 118, no. 4, pp. 2365–2372, October 2005.

[4] Y. Jin, J. M. F. Moura, Y. Jiang, D. Stancil, and A. Cepni, "Time reversal detection in clutter: additional experimental results," *IEEE Transactions on Aerospace and Electronic Systems*, to appear.

[5] J. Gomes and V. Barroso, "Doppler compensation in underwater channels using time-reversal arrays," in *ICASSP'03, IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. V. Hong Kong, China: IEEE, April 2003, pp. 81–84.

[6] M. Skolnik, *Radar Handbook*, 3rd ed. New York, NY: McGraw Hill, 2008.

[7] B. Friedlander and A. Zeira, "Detection of broadband signals in frequency and time dispersive channels," *IEEE Transactions on Signal Processing*, vol. 44, no. 7, pp. 1613–1622, July 1996.

[8] L. L. Scharf, *Statistical Signal Processing: Detection, Estimation, and Time Series Analysis*. Reading, MA: Addison-Wesley Publishing Company, 1991.