TIME REVERSAL TARGET CLASSIFICATION FROM SCATTERED RADIATION

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ABSTRACT

This paper proposed the M-ary hypothesis testing algorithm for classifying radar backscatter signals from hidden targets in a rich scattering environment using time reversal. The target recognition algorithm is to be designed to distinguish measurements of the radar backscatter from an unknown object as belonging to one of a set of M classes.

The proposed time reversal target classifier is, in essence, a correlator that calculates the cross-correlation of the normalized target signature waveforms with a data dependent quantity obtained from measurements. The algorithm requires *a priori* empirical statistical knowledge of the scattering channel, which is dependent of configurations of the scatterers in the environment. By incorporating time reversal, the proposed algorithm provides a significant performance improvement compared with the conventional method. Proof of concept is provided using electromagnetic data collected in a laboratory environment.

***Index Terms*—** Time Reversal, Target Classification, M-ary Hypothesis Testing, Regularization

1. INTRODUCTION AND MOTIVATION

One of the primary functions of modern radar systems is target classification. Based on the received backscattered energy, the radar system determines which of M pre-specified body shape or size is most consistent with the scattered field data. In this sense, target classification may be viewed as a crude form of inverse scattering.

The target recognition algorithm is to be designed to distinguish measurements of the radar backscatter from an unknown object as belonging to one of a set of M classes, each corresponding to a particular target with a distinct radar signature waveform. The conventional target classification scenario assumes that there exists a direct path between the target and the radar receiver. The line of sight condition enables matched filtering processing in that the received signal is a (complex) scaled version of the target signature. However, the line of sight assumption may not be valid in many practical scenarios.

In a rich scattering environment, the electromagnetic scattering mechanism is very complex due to the dispersive nature of the extended targets as well as the surrounding medium. From an engineering science point of view, the backscattered signal, excluding the incident wave, can be characterized as the product of the target signature and the channel transfer function in frequency domain. The coupling of target response and the dispersive rich scattering environment makes it a challenging problem for target classification. Furthermore, when the target is hidden by an object that blocks the direct illumination from the radar transmitter, the radar receiver must rely on returned energies due to secondary reflections from the surrounding scatterers.

Time reversal is a technique that utilizes the rich scattering to enhance target detectability and resolution [1, 2]. In general, the returned backscatter signals from the hidden targets are relatively weak. Time reversal in a rich scattering environment is expected to adapt the transmission waveform to the target response and to enhance the target signal returns.

In this paper, we propose an M-ary hypothesis testing algorithm for discriminating between a finite number of targets by appropriately processing the backscattered waveforms in a rich scattering environment using time reversal. In this paper, we assume that each target is characterized by a signature waveform with a distinct resonance frequency. In the theory of scattering [3], the resonance frequency refers to the illumination region where the wavelength of the incident field is of the same order of the size of the target. Illuminating targets in the resonance region is generally considered most efficacious for target shape and size classification[4]. Such studies are of theoretical and practical importance, particularly for wideband systems where the response of the extended targets shows frequency dependency [5].

Target recognition utilizing time reversal provides significant performance advantage relative to conventional matched filter processing. Time reversal shows advantages because it (1) shortens the scattering Green function; and (2) enhances the target scattered field by resonance region matched illumination. We demonstrate our algorithm using electromagnetic data collected in a laboratory environment.

2. PROBLEM FORMULATION

2.1. Data Model

We examine the target recognition problem when the target function exhibits a radar frequency dependent behavior. There

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are well established reasons for this phenomenon, for example, multipath effects and resonance in cavities. We start with the data model in the frequency domain. Suppose that we have M targets with distinct signature $f_1(\omega), \cdots, f_m(\omega)$. The *target radar signature* $f_m(\omega)$ represents a frequency-dependent reflectivity for the m-th target which, in general, is a complex function. Each signature $f_m(\omega)$ contains a phase function and a magnitude function. Using a discrete frequency representation, we have a collection of discrete frequency samples of the signature $f_m(\omega_0), \cdots, f_m(\omega_{Q-1})$ over a frequency range $\omega_0, \cdots, \omega_{Q-1}$ with the frequency spacing of $\Delta\omega = \frac{\omega_{Q-1}-\omega_0}{Q-1}$. We further define $E_m = \frac{1}{Q}\sum_{q=0}^{Q-1}|f_m(\omega_q)|^2$ as the total energy of the m-th radar target in the observing frequency range. We also introduce

$$\overline{f}_m(\omega_q) = \frac{1}{\sqrt{E_m}} f_m(\omega_q) \tag{1}$$

which we henceforth refer to as the normalized m-th radar target signature, where $\frac{1}{Q}\sum_{q=0}^{Q-1}|\overline{f}_m(\omega_q)|^2 = 1$.

Let $S(\omega_q)$ be a wideband probing signal at angular frequency ω_q. Let $G_{c+t}(\omega_q)$ and $G_c(\omega_q)$ denote the Green's function of the channel when both target and clutter are present, and the Green's function of the channel when only clutter are present, respectively, we then define

$$G(\omega_q) = G_{c+t}(\omega_q) - G_c(\omega_q), \forall q \tag{2}$$

as the Green's function due to the presence of the target. The l-th returned data snapshot due to the existence of the m-th target has the following frequency response

$$Y_l(\omega_q) = S(\omega_q) f_m(\omega_q) G(\omega_q) + V_l(\omega_q). \tag{3}$$

In the above model, $G(\omega_q)$ is the Green's function of the medium at ω_q. $V_l(\omega_q)$ is the additive complex Gaussian noise. It is assumed that $V_l(\omega_q)$ is independent of $S(\omega_q)$, $f_m(\omega_q)$ and $G(\omega_q)$. Time reversal, or phase conjugation in frequency domain, shortens the multipath channel. The received time-reversed signal is given by

$$\begin{aligned} X_l(\omega_q) &= k_l Y_l^*(\omega_q) f_m(\omega_q) G(\omega_q) + W_l(\omega_q) &(4)\\ &= k_l S^*(\omega_q)|f_m(\omega_q)|^2|G(\omega_q)|^2 + \\ &\quad k_l f_m(\omega_q) G(\omega_q) V_l^*(\omega_q) + W_l(\omega_q), &(5)\end{aligned}$$

where k_l is the energy normalization factor

$$k_l = \sqrt{\frac{E_s}{E_{y_l}}} = \sqrt{\frac{\sum_{q=0}^{Q-1}|S(\omega_q)|^2}{\sum_{q=0}^{Q-1}|Y_l(\omega_q)|^2}}. \tag{6}$$

The average transmission energy is $E_s = \frac{1}{Q}\sum_{q=0}^{Q-1}|S(\omega_q)|^2$.

We re-write Eqn. (3), (5) in a matrix-vector form as follows:

$$\begin{aligned} \mathbf{y}_l &= [Y_l(\omega_0), \cdots, Y_l(\omega_{Q-1})]^T &(7)\\ \mathbf{x}_l &= [X_l(\omega_0), \cdots, X_l(\omega_{Q-1})]^T &(8)\\ \mathbf{g} &= [G(\omega_0), \cdots, G(\omega_{Q-1})]^T &(9)\\ \mathbf{\Sigma}_{l,m} &= [k_l Y_l^*(\omega_0) f_m(\omega_0), \cdots, \\ &\quad k_l Y_l^*(\omega_{Q-1}) f_m(\omega_{Q-1})]^T &(10)\\ \mathbf{w}_l &= [W_l(\omega_0), \cdots, W_l(\omega_{Q-1})]^T &(11)\\ \mathbf{v}_l &= [V_l(\omega_0), \cdots, V_l(\omega_{Q-1})]^T &(12)\\ \mathbf{\Gamma}_m &= [S(\omega_0) f_m(\omega_0), \cdots, \\ &\quad S(\omega_{Q-1}) f_m(\omega_{Q-1})]^T, &(13)\end{aligned}$$

where $\mathbf{w}_l$ and $\mathbf{v}_l$ are additive complex Gaussian noise vector with statistics

$$\begin{aligned} \mathbf{w}_l &\sim \mathcal{CN}(0, \sigma_w^2 \mathbf{I}_Q) &(14)\\ \mathbf{v}_l &\sim \mathcal{CN}(0, \sigma_v^2 \mathbf{I}_Q). &(15)\end{aligned}$$

In this work, we assume that the noise variance for $\mathbf{v}_l$ and $\mathbf{w}_l$ for $l = 1, \cdots, L$ are the same, i.e., $\sigma_v^2 = \sigma_w^2 = \sigma^2$.

2.2. M-ary Hypothesis Testing

Next, we organize the measurement pair $(\mathbf{y}_l^*, \mathbf{x}_l)$ into a data vector as follows:

$$\mathbf{z}_l = \begin{bmatrix} \mathbf{y}_l^* \\ \mathbf{x}_l \end{bmatrix} = \begin{bmatrix} \mathbf{\Gamma}_m^* \mathbf{g}^* + \mathbf{v}_l^* \\ \mathbf{\Sigma}_{l,m}\mathbf{g} + \mathbf{w}_l \end{bmatrix}, \quad l = 1, \cdots, L. \tag{16}$$

Let $\mathbf{z}$ be the collection of L data snapshots

$$\mathbf{z} = [\mathbf{z}_1^T, \cdots, \mathbf{z}_L^T]^T. \tag{17}$$

The target detection and classification algorithm is based on a sequence of hypothesis tests in which each hypothesis corresponds to a particular radar target signature. We seek a decision rule $\phi(\mathbf{z})$ that takes on values in the set $\{1, 2, \cdots, M\}$ where $\phi(\mathbf{z}) = m$ corresponds to selection of hypothesis $\mathbb{H}_m$. For the purpose of simplicity, we further assume that the occurrence of m-th target is equally likely.

For the case where $\mathbf{g}$ is known, such a decision rule is $\phi(\mathbf{z}) = m$ whenever the likelihood of the m-th target exceeds the likelihood for any other hypothesis: [6]

$$\phi(\mathbf{z}) = m, \quad \text{whenever} \quad p_m(\mathbf{z}) > \max_{n \neq m} p_n(\mathbf{z}) \tag{18}$$

where $p_m(\cdot)$ is the probability density function if the m-th target exists.

In this work, $\mathbf{g}$ is unknown. Thus a generalized hypothesis test is employed and $\mathbf{g}$ needs to be estimated from the measurements. Furthermore, a careful inspection of (3) and (5) reveals that the target signature waveform $f_m(\omega_q)$ to be classified and the unknown channel Green's function $G(\omega_q)$ are coupled, which appears in the form of a product $f_m(\omega_q)G(\omega_q)$. Some constraints on $\mathbf{g}$ are necessary to distinguish $f_m(\omega_q)$ from the unknown $G(\omega_q)$. To this end, we make the following modifications on the data model. First, similar to the normalized target signature waveform given in (1), we may define the following quantities:

$$\overline{\mathbf{g}}_m = [\overline{G}_m(\omega_0), \cdots, \overline{G}_m(\omega_{Q-1})]^T, \quad (19)$$
$$\overline{G}_m(\omega_q) = \sqrt{E_m} G(\omega_q), \quad m = 1, \cdots, M. \quad (20)$$

Thus, without changing the function values, (3) and (5) can be re-written in the following format for $l = 1, \cdots, L$:

$$Y_l(\omega_q) = S(\omega_q)\overline{f}_m(\omega_q)\overline{G}(\omega_q) + V_l(\omega_q), \quad (21)$$
$$X_l(\omega_q) = k_l S^*(\omega_q)|\overline{f}_m(\omega_q)|^2|\overline{G}(\omega_q)|^2 + k_l\overline{f}_m(\omega_q)\overline{G}(\omega_q)V_l^*(\omega_q) + W_l(\omega_q). \quad (22)$$

Thus, we will estimate $\overline{\mathbf{g}}_m, m = 1, \cdots, M$ rather than $\mathbf{g}$ directly. Second, a regularization term on $\overline{\mathbf{g}}_m$ will be introduced. One method for recovering $\overline{\mathbf{g}}_m$ from $\mathbf{z}$ is to define $\widehat{\overline{\mathbf{g}}}_m$

$$\widehat{\overline{\mathbf{g}}}_m = [\widehat{\overline{G}}_m(\omega_0), \cdots, \widehat{\overline{G}}_m(\omega_{Q-1})]^T, \quad (23)$$

the estimate of the Green's function vector, as the solution to the nonlinear least squares problem

$$\widehat{\overline{\mathbf{g}}}_m = \arg\min \frac{1}{L}\sum_{l=1}^{L} \|\mathbf{z}_l - \mathbf{z}_l^t(\overline{\mathbf{g}}_m)\|^2_{\mathbf{\Omega}_l^{-1}} + \alpha_m^2\|\mathbf{g}\|^2_{\mathbf{\Phi}^{-1}} \quad (24)$$

where $\|\mathbf{x}\|^2_{\mathbf{A}} = \mathbf{x}^H\mathbf{A}\mathbf{x}$. The block diagonal matrix $\mathbf{\Omega}_l$ as a function of $\overline{\mathbf{g}}_m$, is generally a matrix whose entries reflect the noise levels present in the measurements, $\mathbf{z}_l$,

$$\mathbf{\Omega}_l = \mathrm{diag}[\Omega_l(0), \cdots, \Omega_l(Q-1)] \quad (25)$$

where

$$\Omega_l(q) = \begin{bmatrix} \sigma^2 & k_l\overline{f}_m^*(\omega_q)\overline{G}^*(\omega_q)\sigma^2 \\ k_l\overline{G}(\omega_q)\overline{f}_m(\omega_q)\sigma^2 & k_l^2|\overline{G}(\omega_q)|^2|\overline{f}_m(\omega_q)|^2\sigma^2 + \sigma^2 \end{bmatrix} \quad (26)$$

The vector $\mathbf{z}_l^t$ as a function of $\overline{\mathbf{g}}_m$, is defined as follows:

$$\mathbf{z}_l^t = \left[[\mathbf{z}_l^t(0)]^T, \cdots, [\mathbf{z}_l^t(Q-1)]^T\right]^T, \quad (27)$$

where

$$\mathbf{z}_l^t(q) = \begin{bmatrix} S^*(\omega_q)\overline{f}_m^*(\omega_q)\overline{G}^*(\omega_q) \\ k_l|\overline{f}_m(\omega_q)|^2|\overline{G}(\omega_q)|^2 S^*(\omega_q) \end{bmatrix}. \quad (28)$$

The $Q \times Q$ positive definite matrix $\mathbf{\Phi}$ is employed to regularize the problem, where

$$\mathbf{\Phi} = \mathrm{diag}[\phi_0^2, \cdots, \phi_{Q-1}^2]. \quad (29)$$

The scalar α_m^2 is defined by

$$\alpha_m^2 = \frac{E_m}{\sigma^2} \quad (30)$$

as the m-th target signature power to noise ratio. In fact, we can re-write the second term of (24) as follows:

$$\alpha_m^2\|\mathbf{g}\|^2_{\mathbf{\Phi}^{-1}} = \sum_{q=0}^{Q-1}\frac{|\overline{G}_m(\omega_q)|^2}{\phi_q^2\sigma^2} = \|\overline{\mathbf{g}}_m\|^2_{(\sigma^2\mathbf{\Phi})^{-1}}. \quad (31)$$

We note that the nonlinear least squares formulation admits an interpretation in the context of optimal statistical estimation. Eqn. (24), with the exception of the scalar α_m^2, is equivalent to the maximum *a posteriori* (MAP) estimate of $\mathbf{g}$ given $\mathbf{z}_l$ assuming that

$$\mathbf{z}_l \sim \mathcal{CN}(\mathbf{z}_l^t, \mathbf{\Omega}_l), \quad (32)$$
$$\mathbf{g} \sim \mathcal{CN}(0, \mathbf{\Phi}), \quad \mathbf{\Phi} = \mathrm{diag}[\phi_0^2, \cdots, \phi_{Q-1}^2]. \quad (33)$$

In our case, α_m in (24) is introduced as a weighting factor to modify the MAP estimate according to the target signature power relative to the noise power. Note that the performance of the classifier is affected by the choice of the regularization term $\mathbf{\Phi}$. The statistical knowledge of the scattering environment is essential to classify the radar targets. In a dense and rich scattering environment without direct line of sight, the empirical distribution of the target channel response resembles, for example, complex Gaussian [2]. Here we assume that the empirical distribution of the scattering environment can be estimated.

Let $p_m(\mathbf{z}|\mathbf{g})$ be the probability density function of the m-th target conditioned on $\mathbf{g}$ and $k_l, l = 1, \cdots, L$ as follows:

$$\begin{aligned} p_m(\mathbf{z}|\mathbf{g}) &= \prod_{l=1}^{L}\frac{1}{\pi|\mathbf{\Omega}_l|}\exp\left\{-(\mathbf{z}_l - \mathbf{z}_l^t)^H\mathbf{\Omega}_l^{-1}(\mathbf{z}_l - \mathbf{z}_l^t)\right\} \\ &= \prod_{l=1}^{L}\prod_{q=0}^{Q-1}\frac{1}{\pi\sigma^4}\exp\Bigg\{-\frac{1}{\sigma^2}(|Y_l(\omega_q)|^2k_l^2\times \\ &\quad |\overline{f}_m(\omega_q)|^2 + |S(\omega_q)|^2|\overline{f}_m(\omega_q)|^2)|\overline{G}_m(\omega_q)|^2 \\ &\quad +\frac{2}{\sigma^4}\Re\{[k_lX_l^*(\omega_q)Y_l^*(\omega_q)\overline{f}_m(\omega_q)\sigma^2 \\ &\quad +\sigma^2Y_l^*(\omega_q)\overline{f}_m(\omega_q)S(\omega_q)]\overline{G}_m(\omega_q)\} \\ &\quad -\frac{1}{\sigma^4}\left(|X_l(\omega_q)|^2\sigma^2 + |Y_l(\omega_q)|^2\sigma^2\right)\Bigg\}. \end{aligned} \quad (34)$$

Taking the logarithmic of (34), discarding the constant terms, and multiplying $-\frac{1}{L}$ yields $-\frac{1}{L}\ln p_m(\mathbf{z}|\mathbf{g})$ Hence, the optimization problem (24) is equivalent to seeking the minimum of the sum of $-\frac{1}{L}\ln p_m(\mathbf{z}|\mathbf{g})$ and (31). A common practice of calculating the maximum *a posteriori* estimate of $\overline{G}_m(\omega_q)$ is by taking the first derivative of the sum of $-\frac{1}{L}\ln p_m(\mathbf{z}|\mathbf{g})$ and (31). This leads to the following equivalent optimization problem of (24):

$$\widehat{\overline{\mathbf{g}}}_m = \arg\min_m\left\{-\frac{1}{L}\ln p_m(\mathbf{z}|\mathbf{g}) + \|\overline{\mathbf{g}}_m\|^2_{(\sigma^2\mathbf{\Phi})^{-1}}\right\}. \quad (35)$$

Notice that for complex number x, $\frac{\partial}{\partial x^*}|x|^2 = x$ [7]. We then calculate the maximum *a posteriori* estimate of $\overline{G}_m(\omega_q)$

by taking the first derivative of (35) with respect to $[\overline{G}_m(\omega_q)]^*$. We obtain the following estimate:

$$\widehat{\overline{G}}_m(\omega_q) = \frac{\overline{f}^*_m(\omega_q)\frac{1}{L}\sum_{l=1}^{L}\left[Y_l(\omega_q)S^*(\omega_q) + k_l Y_l(\omega_q)X_l(\omega_q)\right]}{|\overline{f}_m(\omega_q)|^2\frac{1}{L}\sum_{l=1}^{L}\left[|S(\omega_q)|^2 + |Y_l(\omega_q)|^2 k_l^2\right] + \frac{1}{\phi_q^2}}. \quad (36)$$

Hence, the generalized M-ary hypothesis is employed in which $\overline{\mathbf{g}}_j$ is replaced by the maximum *a posteriori*-type estimate $\widehat{\overline{\mathbf{g}}}_j$ for $j = 1, \cdots, M$ in (36) and (23) as follows:

$$\mathbb{H}_m : \quad \phi(\mathbf{z}) = m \text{ with}$$

$$m = \arg\min_{j=1,\cdots,M} \frac{1}{L}\sum_{l=1}^{L} \|\mathbf{z}_l - \mathbf{z}_l^t(\widehat{\overline{\mathbf{g}}}_j)\|^2_{\mathbf{\Omega}_l^{-1}(\widehat{\overline{\mathbf{g}}}_j)}. \quad (37)$$

The decision rule (37) to classify m-th target can be simplified as $\phi(\mathbf{z}) = m$ when

$$\max_{j=1,\cdots,M} \sum_{q=0}^{Q-1} |\overline{f}_j(\omega_q)|^2 \left|\frac{1}{L}\sum_{l=1}^{L}(k_l X_l^*(\omega_q)Y_l^*(\omega_q) + Y_l^*(\omega_q)S(\omega_q))\right|^2 \times \frac{|\overline{f}_j(\omega_q)|^2\frac{1}{L}\sum_{l=1}^{L}\left(|Y_l(\omega_q)|^2 k_l^2 + |S(\omega_q)|^2\right) + \frac{2}{\phi_q^2}}{\left||\overline{f}_j(\omega_q)|^2\frac{1}{L}\sum_{l=1}^{L}\left[|S(\omega_q)|^2 + |Y_l(\omega_q)|^2 k_l^2\right] + \frac{1}{\phi_q^2}\right|^2} \quad (38)$$

To gain insight of the above decision rule, we let

$$\mathbf{a}_j = [A_j(\omega_0), \cdots, A_j(\omega_{Q-1})]^T \quad (39)$$

$$A_j(\omega_q) = |\overline{f}_j(\omega_q)|^2 \quad (40)$$

be the collection of magnitude squared j-th normalized discrete target signature waveform and let

$$\widehat{B}_{j,m}(q) = \left|\frac{1}{L}\sum_{l=1}^{L}(k_l X_l^*(\omega_q)Y_l^*(\omega_q) + Y_l^*(\omega_q)S(\omega_q))\right|^2 \frac{|\overline{f}_j(\omega_q)|^2\frac{1}{L}\sum_{l=1}^{L}\left(|Y_l(\omega_q)|^2 k_l^2 + |S(\omega_q)|^2\right) + \frac{2}{\phi_q^2}}{\left(|\overline{f}_j(\omega_q)|^2\frac{1}{L}\sum_{l=1}^{L}\left(|S(\omega_q)|^2 + |Y_l(\omega_q)|^2 k_l^2\right) + \frac{1}{\phi_q^2}\right)^2} \quad (41)$$

$$\widehat{\mathbf{b}}_{j,m} = [\widehat{B}_{j,m}(\omega_0), \cdots, \widehat{B}_{j,m}(\omega_{Q-1})]^T \quad (42)$$

be the estimate of the m-th target signature waveform given the knowledge of the j-th target signature. We can rewrite (37), (38) as follows

$$\phi(\mathbf{z}) = m \quad \text{when} \quad m = \arg\max_{j=1,\cdots,M} \langle \mathbf{a}_j, \widehat{\mathbf{b}}_{j,m}\rangle. \quad (43)$$

The decision rule (43) indicates that the classifier is, in essence, a correlator between the magnitude-squared of j-th normalized target signature waveform $\overline{f}_j(\omega_q)$ and the estimate of the m-th target signature waveform given the knowledge of the j-th target signature.

In summary, the design of the classifier consists of three steps: (1). compute the maximum *a posteriori* (MAP) estimate of $\overline{G}_j(\omega_q)$; (2). insert the obtained estimate into the decision rule (43); (3). declare the existence of the m-th target if its decision value is the maximum among $j \in [1, \cdots, M]$ decision values given by (43).

2.3. Matched Filter Classifier (MF)

We consider a direct correlation classifier where the target signature waveform frequency response vector correlates with the measured backscatters. It is assumed that the propagation delay τ due to the target is an unknown parameter and is to be estimated. The estimation is carried out by searching over a range $\tau \in [0, \frac{2\pi}{\Delta f}]$, where $\Delta f = \frac{f_{Q-1}-f_0}{Q-1}$. Let $\mathbf{y} = [\mathbf{y}_1^T, \cdots, \mathbf{y}_{L'}]^T$ be the collection of data vectors. Thus we obtain the following correlation classifier: $\phi(\mathbf{y}) = m$ for

$$\max_{j=1,\cdots,M} \max_{\tau} \Re\left(\frac{1}{\sqrt{\sigma^2 E_{f_j}}}\sum_{q=0}^{Q-1}[S(\omega_q)f_j(\omega_q)e^{-j\omega_q\tau}]^* \sum_{l'=1}^{L'} Y_{l'}(\omega_q)\right), \quad (44)$$

where $L' = 2L$, $\Re(\cdot)$ is the real part of complex numbers. $E_{f_j} = \frac{1}{Q}\sum_{q=0}^{Q-1}|S(\omega_q)f_j(\omega_q)|^2$. We notice that (44) implements a matched filer.

3. ELECTROMAGNETIC EXPERIMENTS

3.1. Frequency dependent target design

We design frequency dependent targets using aluminum foam slices. Each aluminum foam slice is considered as a dipole antenna with resonance frequency $f_R = \frac{c}{2l}$, where $l = \frac{\lambda}{2}$ is the length of the foam slice. A careful choice of the aluminum slice length ensures that each target has a distinct resonance frequency. A total of 25 aluminum foam slices are glued vertically on a foam board and placed on a 25.5 cm by 25.5 cm grid. Each grid has the size of 5.1 cm by 5.1 cm. Thus, each foam board is considered as a frequency dependent target. The upper figure in Fig. 1 shows the measured (smoothed) target signature waveforms.

3.2. Electromagnetic measurement

The measurement are performed in $4-6$ GHz frequency domain in a controlled laboratory environment. Two horn antennas, one transmits and one receives, are connected to an Agilent vector network analyzer (VNA). A total of 201 frequency points are recorded. The probing signal, denoted by $S(\omega_q)$, has 2 GHz bandwidth with center frequency of 5 GHz.

When measuring the backscatter returns, each target is covered completely by a white board without a clear line of sight in the view of transmitter and receiver antennas. PVC pipes are placed behind the targets to create a rich scattering

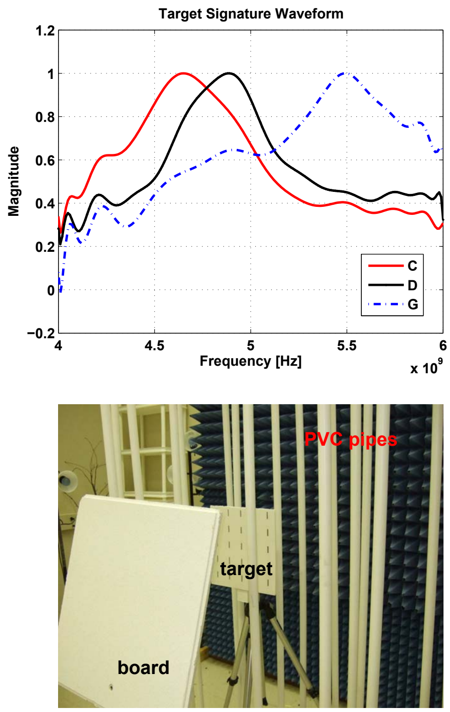

Fig. 1. Upper figure: target signature waveform; Lower figure: picture of the experimental setup.

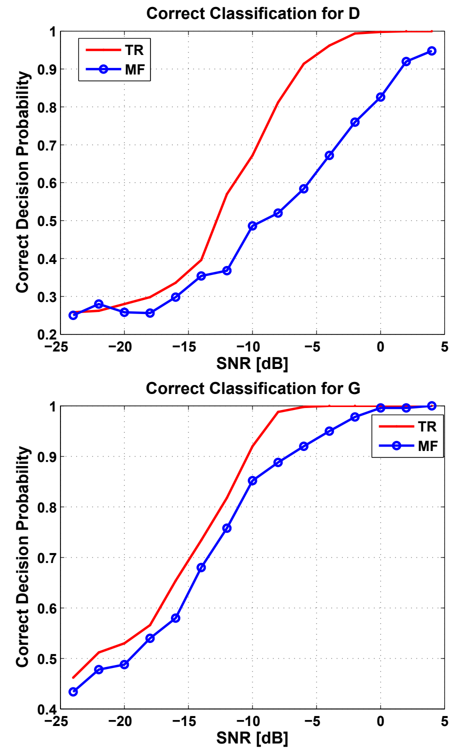

Fig. 2. Probability of correct classification curve for the time reversal (TR) classifier and the matched filter (MF) classifier.

environment. Thus we can only rely on the backscatters reflected off the surrounding scatterers to detect the target. We first measure the frequency response with the target and the board. Then we place scatterers around the target. The difference of the consecutive measurements is $f_m(\omega_q)G(\omega_q)$. The computer generated artificial noise is inserted into the measurement.

4. PERFORMANCE RESULTS

Fig. 2 shows the correct classification probability for target D and target G. The two targets appear to have different peak resonance frequencies and total energy. For target D, the peak frequency is at 4.8 GHz; for target G, the peak frequency is at 5.5 GHz. The time reversal classifier shows better performance than the matched filter classifier.

5. CONCLUSION AND FUTURE WORK

We proposed the M-ary hypothesis testing using time reversal for classifying hidden targets immersed in a rich scattering environment. The designed classifier is affected by a regularization term which depends on the empirical distribution of the Green's function. In this paper, we assume a Gaussian distribution of the rich scattering channel. Future work is to investigate other statistical modeling of the scattering Green's function and its impact on hidden target classification.

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